

MAY/JUNE 2009

**EXAMINATION  
MATHEMATICS**

**Paper 02 – General Proficiency**

*2 hours 40 minutes*

**20 MAY 2009 (a.m.)**

**INSTRUCTIONS TO CANDIDATES**

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

**Examination Materials**

Electronic calculator (non-programmable)

Geometry set

Mathematical tables

Graph paper (provided)

## LIST OF FORMULAE

Volume of a prism  $V = Ah$  where  $A$  is the area of a cross-section and  $h$  is the perpendicular length.

Volume of cylinder  $V = \pi r^2 h$  where  $r$  is the radius of the base and  $h$  is the perpendicular height.

Volume of a right pyramid  $V = \frac{1}{3}Ah$  where  $A$  is the area of the base and  $h$  is the perpendicular height.

Circumference  $C = 2\pi r$  where  $r$  is the radius of the circle.

Area of a circle  $A = \pi r^2$  where  $r$  is the radius of the circle.

Area of trapezium  $A = \frac{1}{2}(a + b)h$  where  $a$  and  $b$  are the lengths of the parallel sides and  $h$  is the perpendicular distance between the parallel sides.

Roots of quadratic equations If  $ax^2 + bx + c = 0$ ,

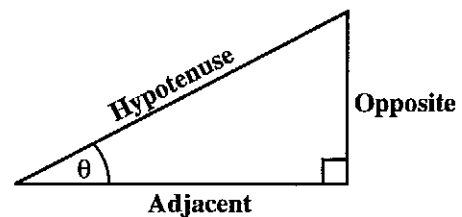
$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric ratios

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



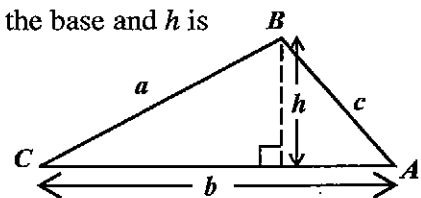
Area of triangle

Area of  $\Delta = \frac{1}{2}bh$  where  $b$  is the length of the base and  $h$  is the perpendicular height

$$\text{Area of } \Delta ABC = \frac{1}{2}ab \sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a + b + c}{2}$$



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

$$(i) \quad \frac{2\frac{2}{3} + 1\frac{1}{5}}{6\frac{2}{5}}$$

giving your answer as a common fraction ( 3 marks)

$$(ii) \quad \sqrt{\left(\frac{0.0256}{25}\right)}$$

giving your answer in standard form. ( 3 marks)

- (b) The basic wage earned by a truck driver for a 40-hour week is \$560.00.

(i) Calculate his hourly rate. ( 1 mark )

For overtime work, the driver is paid one and a half times the basic hourly rate.

(ii) Calculate his overtime wage for 10 hours of overtime. ( 2 marks)

(iii) Calculate the TOTAL wages earned by the truck driver for a 55-hour week. ( 3 marks)

**Total 12 marks**

2. (a) Factorise completely:

(i)  $2ax + 3ay - 2bx - 3by$  ( 2 marks)

(ii)  $5x^2 - 20$  ( 2 marks)

(iii)  $3x^2 + 4x - 15$  ( 2 marks)

- (b) One packet of biscuits costs \$ $x$  and one cup of ice cream costs \$ $y$ .

One packet of biscuits and two cups of ice cream cost \$8.00, while three packets of biscuits and one cup of ice cream cost \$9.00.

(i) Write a pair of simultaneous equations in  $x$  and  $y$  to represent the given information above. ( 2 marks)

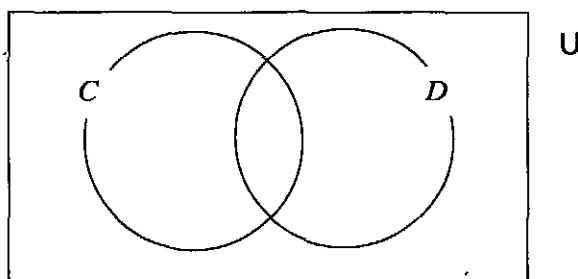
(ii) Solve the equations obtained in (b) (i) above to find the cost of one packet of biscuits and the cost of one cup of ice cream. ( 4 marks)

**Total 12 marks**

3. (a) In a survey of 50 students,  
 23 owned cellular phones  
 18 owned digital cameras  
 $x$  owned cellular phones and digital cameras  
 $2x$  owned neither.

Let  $C$  represent the set of students in the survey who owned cellular phones, and  $D$  the set of students who owned digital cameras.

- (i) Copy and complete the Venn diagram below to represent the information obtained from the survey.

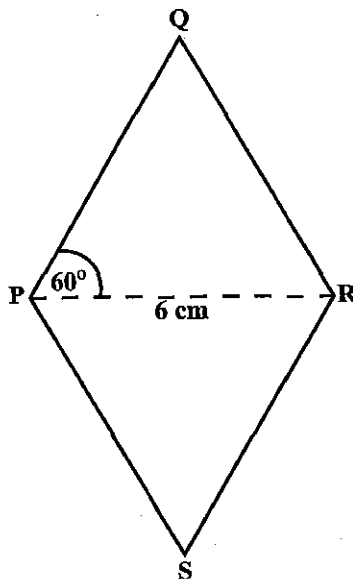


( 2 marks)

- (ii) Write an expression in  $x$  for the TOTAL number of students in the survey. ( 1 mark )

- (iii) Calculate the value of  $x$ . ( 2 marks)

- (b) The diagram below, **not drawn to scale**, shows a rhombus,  $PQRS$ , with the diagonal  $PR = 6$  cm, and the angle  $RPQ = 60^\circ$ .



- (i) Using a ruler, a pencil, and a pair of compasses, construct the rhombus  $PQRS$  accurately. ( 4 marks)
- (ii) Join  $QS$ . Measure and state, in centimetres, the length of  $QS$ . ( 2 marks)

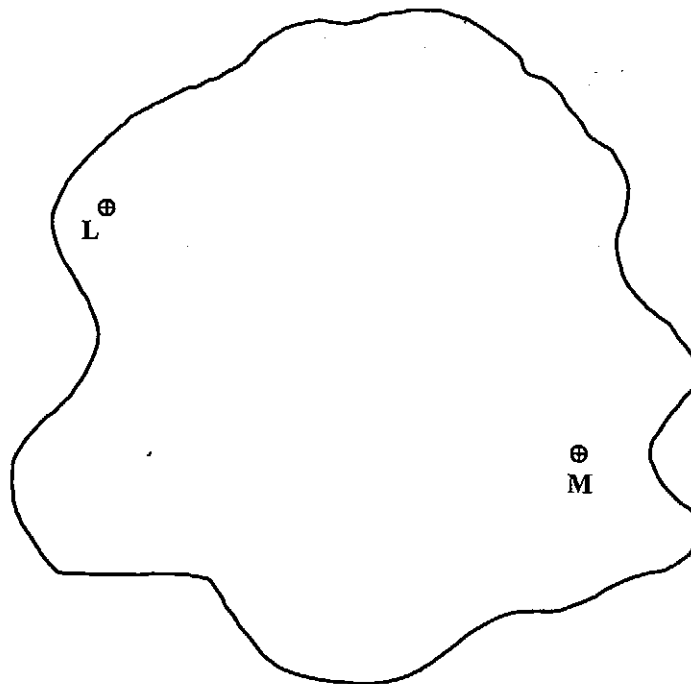
**Total 11 marks**

4. (a) The table below shows two readings taken from an aircraft's flight record.

Time	Distance Travelled (km)
08:55	957
09:07	1083

For the period of time between the two readings, calculate

- (i) the distance travelled in kilometres ( 1 mark )
- (ii) the average speed of the aircraft in km/h. ( 3 marks )
- (b) The map shown below is drawn to a scale of 1:50 000.



- (i) Measure and state, in centimetres, the distance on the map from *L* to *M* along a straight line. ( 2 marks )
- (ii) Calculate the actual distance, in kilometres, from *L* to *M*. ( 2 marks )
- (iii) The actual distance between two points is 4.5 km. Calculate the number of centimetres that should be used to represent this distance on the map. ( 3 marks )

**Total 11 marks**

5. (a) Given that  $f(x) = 2x - 5$  and  $g(x) = x^2 - 31$ , calculate the value of

(i)  $f(-2)$  ( 1 mark )

(ii)  $gf(1)$  ( 2 marks )

(iii)  $f^{-1}(3)$ . ( 2 marks )

(b) Given that  $y = x^2 + 2x - 3$

(i) Copy and complete the table below.

$x$	-4	-3	-2	-1	0	1	2
$y$	5		-3	-4	-3		5

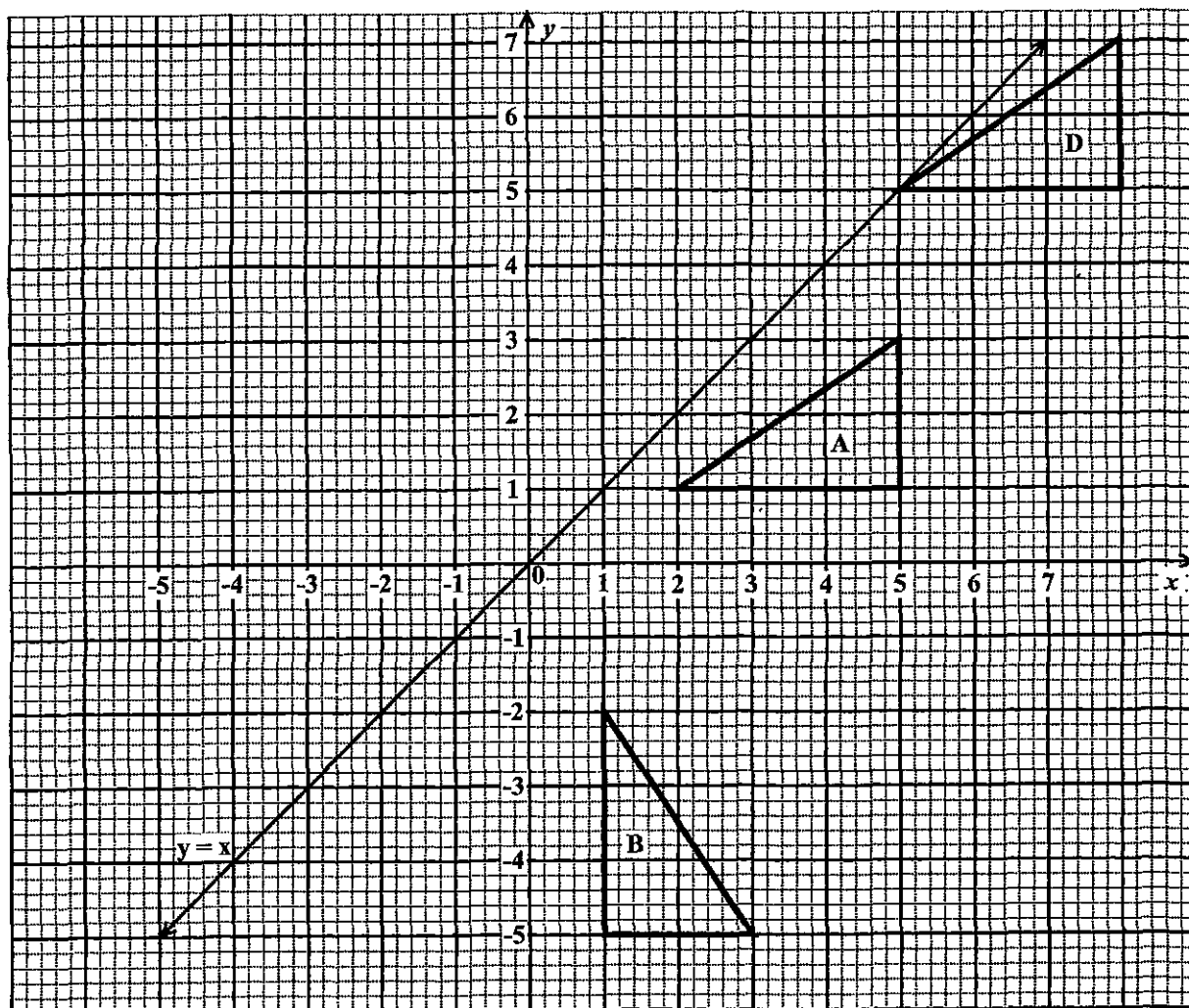
( 2 marks )

(ii) Using a scale of **2 cm to represent 1 unit on the x-axis** and **1 cm to represent 1 unit on the y-axis**, draw the graph of  $y = x^2 + 2x - 3$  for  $-4 \leq x \leq 2$ .

( 5 marks )

**Total 12 marks**

6. The diagram below shows triangles  $A$ ,  $B$  and  $D$ . The line  $y = x$  is also shown.



- (a) Describe, FULLY, the single transformation which maps triangle  $A$  onto
- triangle  $D$  ( 3 marks)
  - triangle  $B$ . ( 3 marks)
- (b) State the coordinates of the vertices of triangle  $C$ , the image of triangle  $A$  after a reflection in the line  $y = x$ . ( 4 marks)

**Total 10 marks**

7. **An answer sheet is provided for this question.**

The table below shows the time, to the nearest minute, that 80 students waited to be served at a school's canteen.

<b>Waiting Time (minutes)</b>	<b>No. of Students</b>	<b>Cumulative Frequency</b>
1 – 5	4	4
6 – 10	7	11
11 – 15	11	22
16 – 20	18	
21 – 25	22	
26 – 30	10	
31 – 35	5	
36 – 40	3	

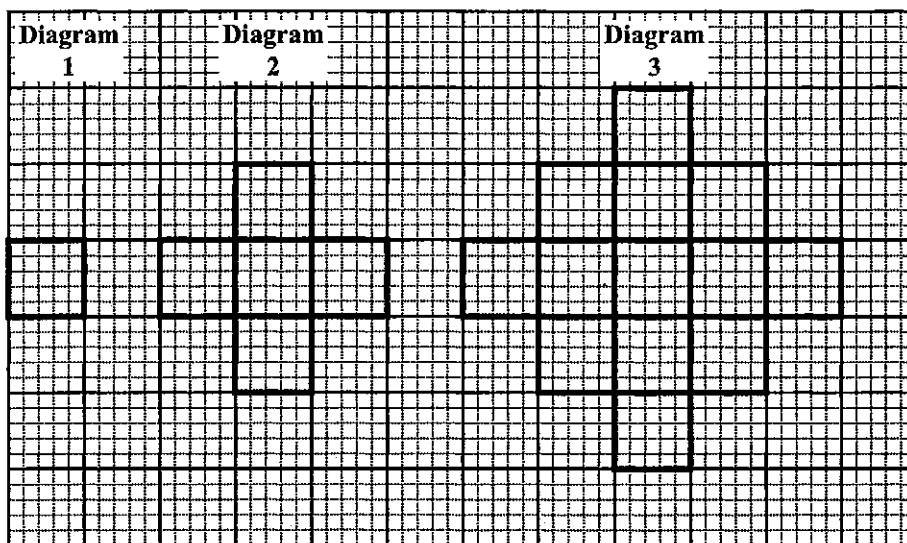
- (a) Copy and complete the table, showing the cumulative frequency. ( 2 marks)
- (b) **On the answer sheet provided**, use the values from your table to complete the cumulative frequency curve. ( 4 marks)
- (c) Use your graph from (b) above to estimate
- (i) the median for the data ( 2 marks)
  - (ii) the number of students who waited for **no more than 29** minutes ( 2 marks)
  - (iii) the probability that a student, chosen at random from the group, waited for **no more than 17** minutes. ( 2 marks)

**Total 12 marks**

**8. An answer sheet is provided for this question.**

The drawings below show the first three diagrams in a sequence. Each diagram in the sequence is obtained by drawing a 1-unit square on each side that forms the perimeter of the previous diagram.

For example, Diagram 2 is obtained by drawing a 1-unit square on each of the four sides of Diagram 1.



**On the answer sheet provided:**

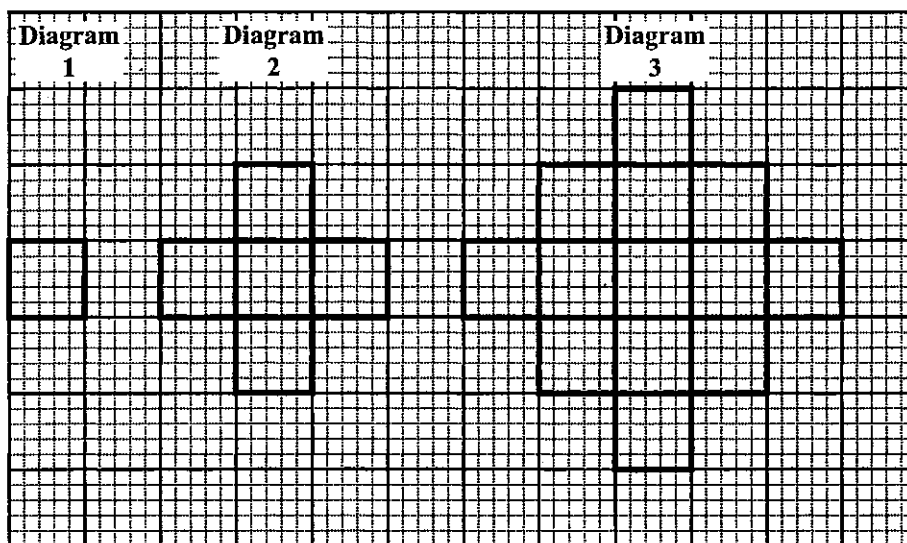
- (a) Draw Diagram 4 in the sequence. ( 2 marks)
- (b) Complete the table by inserting the appropriate values at the rows marked (i), (ii) and (iii). ( 8 marks)

**Total 10 marks.**

**8. An answer sheet is provided for this question.**

The drawings below show the first three diagrams in a sequence. Each diagram in the sequence is obtained by drawing a 1-unit square on each side that forms the perimeter of the previous diagram.

For example, Diagram 2 is obtained by drawing a 1-unit square on each of the four sides of Diagram 1.



**On the answer sheet provided:**

- (a) Draw Diagram 4 in the sequence. ( 2 marks)
- (b) Complete the table by inserting the appropriate values at the rows marked (i), (ii) and (iii). ( 8 marks)

**Total 10 marks**

## SECTION II

Answer TWO questions in this section.

## RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Solve the pair of simultaneous equations
- $$y = 4 - 2x$$
- $$y = 2x^2 - 3x + 1. \quad (4 \text{ marks})$$
- (b) Express  $2x^2 - 3x + 1$  in the form  $a(x + h)^2 + k$ , where  $a$ ,  $h$  and  $k$  are real numbers. (3 marks)
- (c) Using your answer from (b) above, or otherwise, calculate
- (i) the minimum value of  $2x^2 - 3x + 1$  (1 mark)
- (ii) the value of  $x$  for which the minimum occurs. (1 mark)
- (d) Sketch the graph of  $y = 2x^2 - 3x + 1$ , clearly showing
- the coordinates of the minimum point
  - the value of the  $y$ -intercept
  - the values of  $x$  where the graph cuts the  $x$ -axis. (4 marks)
- (e) Sketch on your graph of  $y = 2x^2 - 3x + 1$ , the line which intersects the curve at the values of  $x$  and  $y$  calculated in (a) above. (2 marks)

**Total 15 marks**

10. (a) The owner of a shop wishes to buy  $x$  guitars and  $y$  violins. To satisfy the demands of his customers, the number of violins must be less than or equal to the number of guitars.

(i) Write an inequality to represent this information. ( 1 mark )

The cost of one guitar is \$150 and the cost of one violin is \$300. He has \$4 500 to spend on the purchase of these instruments.

(ii) Write an inequality to represent this information. ( 2 marks )

To get a good bargain, the owner of the shop must buy at least 5 violins.

(iii) Write an inequality to represent this information. ( 1 mark )

- (b) (i) Using a scale of **2 cm on the horizontal axis to represent 5 guitars**, and **2 cm on the vertical axis to represent 5 violins**, draw the graphs of the lines associated with the **THREE** inequalities written in (a) (i), (ii) and (iii) above. ( 4 marks )

(ii) Shade the region on your graph that satisfies all **THREE** inequalities. ( 1 mark )

(iii) State the coordinates of the vertices of the shaded region. ( 2 marks )

- (c) The owner of the shop sells the instruments to make a profit of \$60 on each guitar and \$100 on each violin.

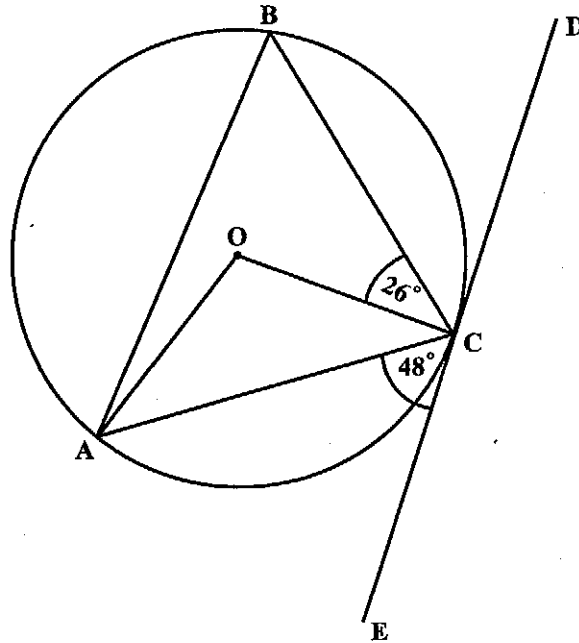
(i) Express the **TOTAL** profit in terms of  $x$  and  $y$ . ( 1 mark )

(ii) Calculate the **maximum** profit. ( 3 marks )

**Total 15 marks**

**GEOMETRY AND TRIGONOMETRY**

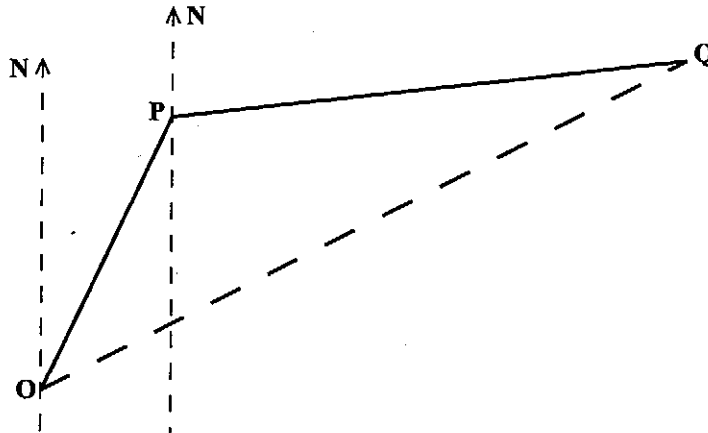
11. (a) The diagram below, **not drawn to scale**, shows a circle, centre  $O$ . The line  $DCE$  is a tangent to the circle. Angle  $ACE = 48^\circ$  and angle  $OCB = 26^\circ$ .



Calculate:

- |       |              |            |
|-------|--------------|------------|
| (i)   | $\angle ABC$ | ( 1 mark ) |
| (ii)  | $\angle AOC$ | ( 1 mark ) |
| (iii) | $\angle BCD$ | ( 1 mark ) |
| (iv)  | $\angle BAC$ | ( 1 mark ) |
| (v)   | $\angle OAC$ | ( 1 mark ) |
| (vi)  | $\angle OAB$ | ( 1 mark ) |

- (b) The diagram below, **not drawn to scale**, shows the positions of two hurricane tracking stations,  $P$  and  $Q$ , relative to a point  $O$ .  $P$  is on a bearing of  $025^\circ$  from  $O$ , and  $OP = 400$  km.  $Q$  is on a bearing of  $080^\circ$  from  $P$  and  $PQ = 700$  km.

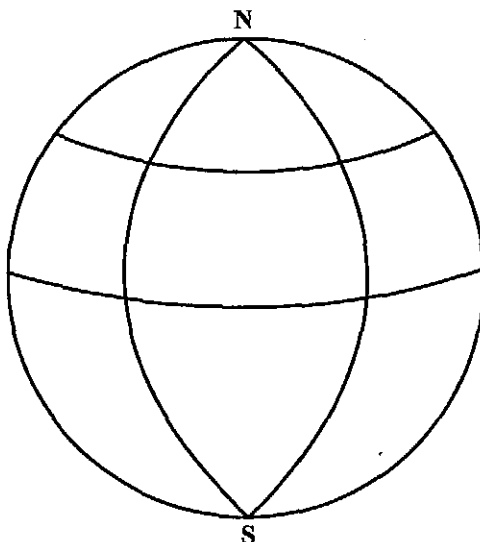


- (i) Copy the diagram above. On your diagram label the angles that show the bearings of  $025^\circ$  and  $080^\circ$ . ( 2 marks)
- (ii) Calculate
- $\angle OPQ$
  - the length, to the nearest kilometre, of  $OQ$
  - the bearing of  $Q$  from  $O$ . ( 7 marks)

**Total 15 marks**

12. In this question, use  $\pi = \frac{22}{7}$ , and assume that the earth is a sphere of radius 6370 km.

The diagram below, **not drawn to scale**, shows a sketch of the earth with the North and South Poles labelled N and S respectively.



Arcs representing circles of longitude  $35^\circ\text{E}$  and  $15^\circ\text{W}$ , and circles of latitude  $0^\circ$  and  $60^\circ\text{N}$  are drawn but not labelled.

- (a) Copy the sketch and
- (i) label the arc which represents:
    - a)  $60^\circ\text{N}$
    - b)  $35^\circ\text{E}$  ( 2 marks)
  - (ii) insert the points:
    - a) J ( $60^\circ\text{N}$ ,  $35^\circ\text{E}$ )
    - b) K ( $60^\circ\text{N}$ ,  $15^\circ\text{W}$ ) ( 2 marks)
- (b) Calculate, **to the nearest kilometre**, the SHORTEST distance from
- (i) J to the North Pole measured along the common circle of longitude ( 3 marks)
  - (ii) J to K measured along the common circle of latitude. ( 4 marks)
- (c) A point H is located 2002 km due south of K along the common circle of longitude.  
Calculate the latitude of H. ( 4 marks)

**Total 15 marks**

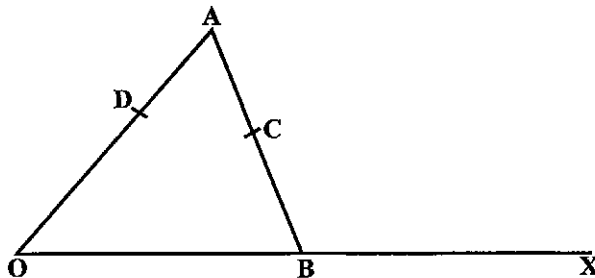
## VECTORS AND MATRICES

13. (a) The points  $A$  and  $B$  have position vectors  $\vec{OA} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$  and  $\vec{OB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$  where  $O$  is the origin  $(0, 0)$ . The point  $G$  lies on the line  $AB$  such that  $AG = \frac{1}{3}AB$ .

Express in the form  $\begin{pmatrix} x \\ y \end{pmatrix}$

- (i) the vectors  $\vec{AB}$  and  $\vec{AG}$  (4 marks)
- (ii) the position vector  $\vec{OG}$ . (2 marks)

- (b) In the diagram below, not drawn to scale,  $B$  is the midpoint of  $OX$ ,  $C$  is the midpoint of  $AB$ , and  $D$  is such that  $OD = 2DA$ . The vectors  $\mathbf{a}$  and  $\mathbf{b}$  are such that  $\vec{OA} = 3\mathbf{a}$  and  $\vec{OB} = \mathbf{b}$ .



- (i) Write in terms of  $\mathbf{a}$  and  $\mathbf{b}$ :
- a)  $\vec{AB}$
- b)  $\vec{AC}$
- c)  $\vec{DC}$
- d)  $\vec{DX}$  (6 marks)
- (ii) State TWO geometrical relationships between  $DX$  and  $DC$ . (2 marks)
- (iii) State ONE geometrical relationship between the points  $D$ ,  $C$ , and  $X$ . (1 mark)

**Total 15 marks**

14. (a) The value of the determinant of  $M = \begin{pmatrix} x & 4 \\ 3 & x \end{pmatrix}$  is 13.

Calculate the values of  $x$ .

( 4 marks)

- (b) The transformation  $R$  is represented by the matrix  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ .

The transformation  $S$  is represented by the matrix  $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ .

- (i) Write a single matrix, in the form  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  to represent the combined transformation  $S$  followed by  $R$ .

( 2 marks)

- (ii) Calculate the image of the point  $(5, -2)$  under the combined transformation in (b) (i) above.

( 3 marks)

- (c) The matrix  $N = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$ .

- (i) Determine the inverse matrix of  $N$ .

( 2 marks)

- (ii) Hence, calculate the value of  $x$  and the value of  $y$  for which

$$\begin{aligned} 3x - y &= 5 \\ 2x + 5y &= 9. \end{aligned}$$

( 4 marks)

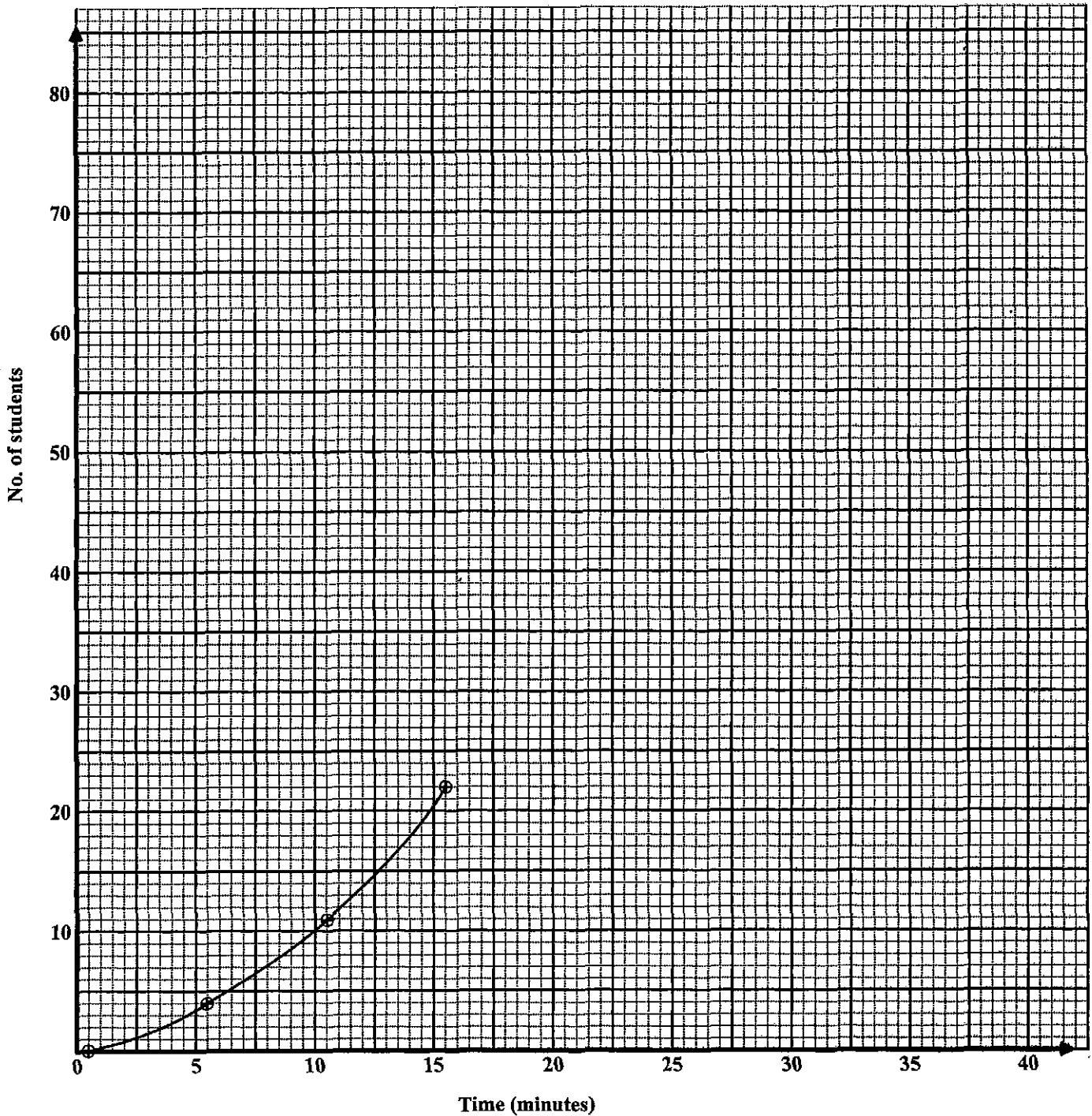
**Total 15 marks**

**END OF TEST**

**EXAMINATION  
MATHEMATICS  
Paper 02 – General Proficiency**

Answer Sheet for Question 7 (b)

Candidate Number .....



**EXAMINATION  
MATHEMATICS  
Paper 02 – General Proficiency**

Answer Sheet for Question 8

Candidate Number .....

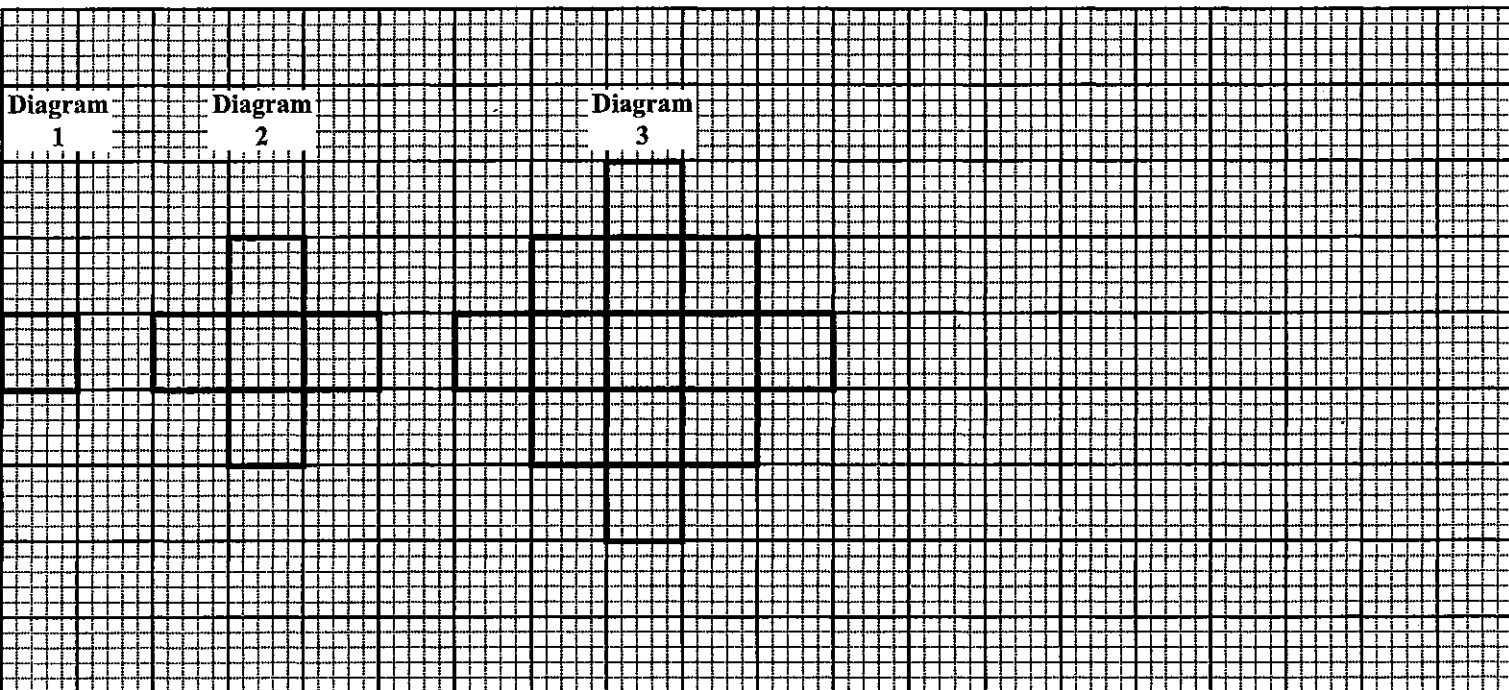


Diagram Number	Number of Unit Squares	Pattern for Calculating Number of Unit Squares
1	1	$1^2 + 0^2$
2	5	$2^2 + 1^2$
3	13	$3^2 + 2^2$
(i) 4	_____	_____ + _____
(ii) 10	_____	_____ + _____
(iii) _____	421	_____ + _____